Lesson 2: Future Worth of $1

Problem 1

Calculate the future value ($FV$) of each present value ($PV$) listed below using the future worth of $1 (FW\$1)$ factors in AH 505:

<table>
<thead>
<tr>
<th></th>
<th>$PV$</th>
<th>$n$ (years)</th>
<th>Annual Rate (%)</th>
<th>$FV = PV \times FW$1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2,500$</td>
<td>5</td>
<td>4</td>
<td>$3,041.63$</td>
</tr>
<tr>
<td>2</td>
<td>$150,000$</td>
<td>10</td>
<td>3</td>
<td>$201,587.46$</td>
</tr>
<tr>
<td>3</td>
<td>$10$</td>
<td>15</td>
<td>4</td>
<td>$18.01$</td>
</tr>
<tr>
<td>4</td>
<td>$250$</td>
<td>10</td>
<td>5</td>
<td>$407.22$</td>
</tr>
<tr>
<td>5</td>
<td>$1,500$</td>
<td>3</td>
<td>10</td>
<td>$1,996.50$</td>
</tr>
</tbody>
</table>

Solution:

To solve, calculate the future value of each present value amount by multiplying the amount by the future worth of $1 factor found in the AH 505 annual compound interest tables for the given interest rate and term.

1. $FV = PV \times FW\$1$
   
   $FV = $2,500 \times 1.216653$ (AH 505, page 25, column 1)
   
   $FV = $3,041.63$

2. $FV = PV \times FW\$1$
   
   $FV = $150,000 \times 1.343916$ (AH 505, page 21, column 1)
   
   $FV = $201,587.46$

3. $FV = PV \times FW\$1$
   
   $FV = $10 \times 1.800944$ (AH 505, page 25, column 1)
   
   $FV = $18.01$

4. $FV = PV \times FW\$1$
   
   $FV = $250 \times 1.628895$ (AH 505, page 29, column 1)
   
   $FV = $407.22$

5. $FV = PV \times FW\$1$
   
   $FV = $1,500 \times 1.331000$ (AH 505, page 49, column 1)
   
   $FV = $1,996.50$
Problem 2

You have a choice of investing in either of two accounts:

- Account A earns an annual rate of 5% with annual compounding
- Account B earns an annual rate of 5.25% at simple interest

You plan to invest $10,000 in one of the accounts today and withdraw the funds 4 years later. In which account should you place your funds?

Solution:
The best choice is the account that will have the most money at the end of the 4 year period. To solve, calculate the future value of each alternative and select the account with the higher future value.

Account A

\[ FV = PV \times FW$1 \ (5\%, \ 4 \ yrs, \ annual) \]
\[ FV = $10,000 \times 1.215506 \ (AH \ 505, \ page \ 29, \ column \ 1) \]
\[ FV = $12,155 \]

Account B

\[ FV = PV + FW$1 \ (5.25\%, \ 4 \ yrs) \ (simple \ interest) \]
\[ FV = $10,000 + ($10,000 \times 5.25\% \times 4 \ yrs) \]
\[ FV = $10,000 + $2,100 \]
\[ FV = $12,100 \]

Because account A produces the higher future value, it is preferred. The annual compounding in account A more than compensates for the higher rate at simple interest in account B.

Problem 3

If you deposit $10,000 in an account that pays an annual interest rate of 7%, how much money will you have at the end of:

1. 10 years?
2. 20 years?
3. 40 years?

Solution:
To solve, calculate the future value of the deposited amount at the end of each term using the future worth of $1 factors in the AH 505 annual compound interest tables with an annual rate of 7%.

After 10 years:

\[ FV = PV \times FW$1 \ (7\%, \ 10 \ yrs, \ annual) \]
\[ FV = $10,000 \times 1.967151 \ (AH \ 505, \ page \ 37, \ column \ 1) \]
\[ FV = $19,672 \]

After 20 years:
• \( FV = PV \times FW\$1 \) (7%, 20 yrs, annual)
  \( FV = \$10,000 \times 3.869684 \) (AH 505, page 37, column 1)
  \( FV = \$38,697 \)

After 40 years:

• \( FV = PV \times FW\$1 \) (7%, 40 yrs, annual)
  \( FV = \$10,000 \times 14.974458 \) (AH 505, page 37, column 1)
  \( FV = \$149,745 \)

**Problem 4**

You deposit \$10,000 today in an account that earns an annual rate of 7.5%. At the end of 5 years, how much interest on interest will the account have earned? *Hint*: The amount of interest on interest is the difference between the future value at compound interest and the future value at simple interest.

**Solution:**

Determine the future value at compound interest:

• \( FV = PV \times FW\$1 \) (7.5%, 5 yrs, annual)
  \( FV = \$10,000 \times 1.435629 \) (AH 505, page 39, column 1)
  \( FV = \$14,356 \)

Determine the future value at simple interest:

• \( FV = PV + (PV \times 7.5\% \times 5 \text{ yrs}) \)
  \( FV = \$10,000 + (\$10,000 \times 7.5\% \times 5 \text{ yrs}) \)
  \( FV = \$10,000 + \$3,750 \)
  \( FV = \$13,750 \)

Amount of interest on interest = \$14,356 - \$13,750 = \$606
Problem 5

You have just purchased an investment property for $400,000. You expect the value of the property to increase at an annual compound interest rate of 8% over the next 10 years. How much do you expect the property to be worth after 10 years?

Solution:
To solve, calculate the expected future value of the investment property using the future worth of $1 factor in the AH 505 annual compound interest tables for 10 years at an annual rate of 8%.

\[ FV = PV \times FW\$1 \ (8\%, \ 10 \ yrs, \ annual) \]
\[ FV = 400,000 \times 2.158925 \ (AH \ 505, \ page \ 41, \ column \ 1) \]
\[ FV = \$863,570 \]

You expect the property to be worth $863,570.

Problem 6

Assuming all cash flows (CF) occur at the end of each year, determine the future values of each of the following two series of cash flows:

<table>
<thead>
<tr>
<th>Series</th>
<th>( CF_1 ) (End Yr 1)</th>
<th>( CF_2 ) (End Yr 2)</th>
<th>( CF_3 ) (End Yr 3)</th>
<th>Annual Rate</th>
<th>FV</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5,000</td>
<td>$5,000</td>
<td>$3,000</td>
<td>5%</td>
<td>[ 1.102500 \times CF ]</td>
</tr>
<tr>
<td>2</td>
<td>$3,000</td>
<td>0</td>
<td>$3,000</td>
<td>4%</td>
<td>[ 1.050000 \times CF ]</td>
</tr>
</tbody>
</table>

Solution:
To solve, calculate the future value of the cash flow at the end of each year using the future worth of $1 factors in the AH 505 annual compound interest tables with an annual rate of 5% for series 1 cash flows and 4% for series 2, and then add the future values of each cash flow in the series. The sum is the future value of the series of cash flows.

\[ FV \text{ of Series 1} (AH \ 505, \ page \ 29, \ column \ 1) \]
\[ FV \text{ of Series 2} (AH \ 505, \ page \ 25, \ column \ 1) \]

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Future Value Factor</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$5,000</td>
<td>1.102500</td>
<td>$5,512.50</td>
</tr>
<tr>
<td>2</td>
<td>$5,000</td>
<td>1.050000</td>
<td>$5,250.00</td>
</tr>
<tr>
<td>3</td>
<td>$3,000</td>
<td>No Compounding</td>
<td>$3,000.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total FV</td>
<td>$13,762.50</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash Flow</th>
<th>Future Value Factor</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$3,000</td>
<td>1.081600</td>
<td>$3,244.80</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>1.040000</td>
<td>0.00</td>
</tr>
<tr>
<td>3</td>
<td>$3,000</td>
<td>No Compounding</td>
<td>$3,000.00</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Total FV</td>
<td>$6,244.80</td>
</tr>
</tbody>
</table>